# A Reference Quality Equation of State for Nitrogen<sup>1</sup>

R. Span<sup>2,5</sup>, E.W. Lemmon<sup>3,4</sup>, R.T Jacobsen<sup>3</sup>, and W. Wagner<sup>2</sup>

Draft, not for publication

<sup>&</sup>lt;sup>1</sup> Paper presented at the Thirteenth Symposium on Thermophysical Properties, June 22-27, 1997, Boulder, Colorado, U.S.A.

<sup>&</sup>lt;sup>2</sup> Lehrstuhl für Thermodynamik, Ruhr-Universität Bochum, D-44780 Bochum, Germany

<sup>&</sup>lt;sup>3</sup> Center for Applied Thermodynamic Studies, University of Idaho, Moscow, ID 83844-1011, U.S.A.

<sup>&</sup>lt;sup>4</sup> Physical and Chemical Properties Division, National Institute of Standards and Technology, 325 Broadway, Boulder, CO 80303, U.S.A.

<sup>&</sup>lt;sup>5</sup> To whom correspondence should be addressed.

## **ABSTRACT**

A new formulation describing the thermodynamic properties of nitrogen has been developed. New data sets are now available which have been used to improve the representation of the p- $\rho$ -T surface of gaseous, liquid and supercritical nitrogen, including the saturated states. New measurements on the speed of sound from spherical resonators have been used to improve the accuracy of caloric properties in gaseous and supercritical nitrogen. Sophisticated procedures for the optimization of the mathematical structure of equations of state and special functional forms for an improved representation of data in the critical region were developed.

The uncertainty of the new reference equation of state ranges from  $\pm 0.02\%$  at pressures less than 30 MPa over nominal temperatures, and up to  $\pm 0.6\%$  at very high pressures. The equation is valid from the triple point to temperatures of 1000 K and pressures up to 2200 MPa. The new formulation yields a reasonable extrapolation behavior up to the limits of chemical stability of nitrogen as indicated by comparison to experimental shock tube data. Constraints regarding the structure of the equation ensure reasonable extrapolated properties up to temperatures and pressures less than 5000 K and 25 GPa.

For typical calibration applications, the new reference equation is supplemented by a simple but also highly accurate formulation, valid only for supercritical nitrogen between 250 and 350 K at pressures up to 30 MPa.

KEY WORDS: caloric properties; density; equation of state; fundamental equation; nitrogen; thermodynamic properties.

## 1. INTRODUCTION

Nitrogen has been one of the most important reference fluids both for tests of physical models and for calibration purposes. The high demands on the availability and accuracy of thermophysical data resulting from this special role led to the development of different reference equations of state which were based on state of the art data sets and correlation techniques available at their time. During the last ten years major improvements have taken place with respect to both of these aspects.

The equation of stated presented by Jacobsen *et al.* [1] has been the accepted standard for the thermodynamic properties of nitrogen since its publication. The equation of state developed in that work was based on the Helmholtz energy. A comprehensive evaluation of the experimental data available up to 1986 was reported by Jacobsen *et al.* [1] Their work superseded the 32 term modified BWR equation of state published by Jacobsen and Stewart [2] in 1973, and the IUPAC international tables of properties for nitrogen published by Angus *et al.* [3] in 1979. Correlations for nitrogen prior to 1973 are reported by Jacobsen and Stewart [2]. The equations of Jacobsen *et al.* and of Stewart and Jacobsen were reported on the International Practical Temperature Scale of 1968 (IPTS-68). The new equation presented here is given on the International Temperature Scale of 1990 (ITS-90).

Many new data sets have become available on which the new nitrogen equation of state presented here is based. These data include the *p*-p-*T* measurements of Nowak *et al*. [4, 5], Klimeck *et al*. [6], Fenghour *et al*. [7], Pieperbeck *et al*. [8], Jaeschke and Hinze [9], Duschek *et al*. [10], and Achtermann *et al*. [11], the isochoric heat capacity data of Magee [12], the speed of sound data of Trusler and Costa Gomez [13], Boyes [14], Ewing and Trusler [15], and Kortbeek *et al*. [16], and the enthalpy data of Owren [17] and Grini [18]. New values of the critical parameters as well as the vapor pressures and coexisting densities were reported by Nowak *et al*. [5]. These new data along with several data sets already used by Jacobsen and Stewart [1] form the basis of the new equation of state for nitrogen. In order to represent the available accurate data within their uncertainty, while maintaining

the least number of coefficients, the new equation was developed using state of the art optimization and multi-property fitting algorithms.

The equation of state is valid for temperatures from the triple point to 1000 K at pressures up to 2200 MPa. The equation extrapolates reasonably up to the limits of chemical stability of nitrogen [19], around 5000 K, at least with respect to basic properties like pressure, fugacity, and enthalpy.

## 2. THE NEW EQUATION

The selected parameters for the critical point and triple point for nitrogen were reported by Nowak *et al.* [5]. These values are

$$T_c = 126.192 \text{ K},$$
 $p_c = 3.3958 \text{ MPa, and}$ 

$$\rho_c = 11.1839 \text{ mol·dm}^{-3}.$$
(1)

The triple-point temperature is 63.151 K and the triple-point pressure is 12.523 kPa [5]. The molar mass is 28.01348 g·mol<sup>-1</sup> [20].

The equation for the ideal gas heat capacity used in this work was developed using the contributions from translation, rotation, and vibration of the nitrogen molecule. This equation is valid from 40 to 5000 K at which point the effects of dissociation are no longer negligible. At temperatures below 40 K, the deviation between heat capacities calculated from the equation used here and those from the statistical model reaches a maximum of 2.7% at 2.4 K. The ideal gas heat capacity passes through a maximum value of 3.597 *R* at this temperature. The equation presented here does not account for this maximum, nor does it account for the decrease in heat capacity to 2.5 *R* near 0 K where the rotational modes are not excited. The equation is expressed as

$$\frac{c_p^0}{R} = 3.5 + 3.066469 \times 10^{-6} T + 4.701240 \times 10^{-9} T^2 - 3.987984 \times 10^{-13} T^3 
+ 1.012941 \frac{u^2 \exp(u)}{\left[\exp(u) - 1\right]^2}$$
(2)

where u is 3364.011 K/T and the gas constant, R, is 8.31451 J·mol<sup>-1</sup>·K<sup>-1</sup>. Since the vibrational relaxation time for nitrogen is very long compared to the period of sound waves, the vibrational modes of the ideal gas heat capacity are not excited by acoustical measurements. These effects become noticeable at temperatures above 300 K. Ideal gas heat capacity values reported from such measurements do not reflect the contribution from vibration. Thus, for calculations of the acoustical speed of sound,  $w_{acs}$ , in nitrogen gas, the thermodynamic speed of sound,  $w_{eos} = \sqrt{(\partial p/\partial \rho)_s}$ , calculated from the equation of state presented here should be corrected according to

$$w_{acs} = w_{eos} \sqrt{\frac{c_v}{c_p} \left(\frac{c_p - c_{vib}}{c_v - c_{vib}}\right)},\tag{3}$$

where  $c_{vib}$  is the exponential term in Eq. (2). Further information is given by Trusler and Costa Gomez [13] and Span *et al.* [21].

The functional form of the new equation of state for nitrogen is explicit in the dimensionless Helmholtz energy,  $\alpha$ , using independent variables of reduced density and temperature. The form of this equation is

$$\frac{a(\rho, T)}{RT} = \alpha(\delta, \tau) = \alpha^{0}(\delta, \tau) + \alpha^{r}(\delta, \tau), \tag{4}$$

where a is the Helmholtz energy,  $\delta = \rho / \rho_c$ ,  $\tau = T_c / T$ , the superscript 0 refers to the ideal gas Helmholtz energy, and the superscript r refers to the residual Helmholtz energy. The ideal gas Helmholtz energy is given by

$$\alpha^{0} = \frac{h_{0}^{0}\tau}{RT_{c}} - \frac{s_{0}^{0}}{R} - 1 + \ln\frac{\delta\tau_{0}}{\delta_{0}\tau} - \frac{\tau}{R} \int_{\tau_{0}}^{\tau} \frac{c_{p}^{0}}{\tau^{2}} d\tau + \frac{1}{R} \int_{\tau_{0}}^{\tau} \frac{c_{p}^{0}}{\tau} d\tau,$$
 (5)

where  $\delta_0 = \rho_0/\rho_c$ ,  $\tau_0 = T_c/T_0$ , and  $\rho_0$  is the ideal gas density at  $T_0 = 298.15$  K and  $p_0 = 0.101325$  MPa. The values of  $h_0^0 = 8670$  J·mol<sup>-1</sup> and  $s_0^0 = 191.5$  J·mol<sup>-1</sup>·K<sup>-1</sup> were chosen to coincide with the values reported by Cox *et al.* [22]. Combining Eqs. (2) and (5), the resulting expression is

$$\alpha^{0} = \ln \delta + 2.5 \ln \tau - 12.76953 - 0.007841630\tau - 1.934819 \times 10^{-4} \tau^{-1} - 1.247742 \times 10^{-5} \tau^{-2} + 6.678326 \times 10^{-8} \tau^{-3} + 1.012941 \ln \left[ 1 - \exp(-26.65788 \tau) \right].$$
(6)

The functional forms of preliminary equations for the residual part of the Helmholtz energy were optimized with the algorithm developed by Setzmann and Wagner [23]. In this algorithm, nonlinear data are linearized (see *e.g.* [24]). To improve the representation of the highly accurate speed of sound data and of the shock tube data available for nitrogen, the final functional form was developed with the nonlinear regression analysis developed by Tegeler *et al.* [25]. Using a combination of linear and nonlinear techniques, this algorithm determines the functional form which yields the best representation of the selected experimental data.

The bank of terms used as basis both for the linear and the nonlinear optimization procedure contained a total of 838 terms, including simple polynomial terms, combinations of polynomials with exponential expressions, and modified Gaussian bell-shaped curve terms which were introduced by Setzmann and Wagner [24] to improve the representation of data in the critical region. Neither the importance of the critical region of nitrogen nor the data situation in this region made it necessary to use the complex nonanalytical terms developed by Span and Wagner [26] for an improved description of caloric data in the immediate vicinity of the critical point.

The residual Helmholtz energy contribution is given by

$$\alpha^{r}(\delta, \tau) = \sum_{k=1}^{6} N_{k} \delta^{i_{k}} \tau^{j_{k}} + \sum_{k=7}^{32} N_{k} \delta^{i_{k}} \tau^{j_{k}} \exp(-\delta^{l_{k}})$$

$$+ \sum_{k=33}^{36} N_{k} \delta^{i_{k}} \tau^{j_{k}} \exp(-\varphi_{k} (\delta - 1)^{2} - \beta_{k} (\tau - \gamma_{k})^{2}),$$
(7)

where the coefficients and parameters of this equation are given in Tables I and II. The coefficients  $N_k$  were determined using the selected data summarized in Table III. In total, 3649 out of the 8298 data points available for p- $\rho$ -T data were used and 1175 out of the 2393 data points available for speed of sound were used. In addition to those data shown in

Table III, 44 isobaric heat capacity data, 14 heat of vaporization data, 203 Joule-Thomson data, and five shock tube data points were used in the fit. Vapor pressure and saturated density data were also used in the fit. Ancillary equations for the vapor pressure, saturated liquid density, and saturated vapor density are given by Nowak *et al.* [5]. The equations have been used without further modifications. The melting line equation of Jacobsen *et al.* [1] was used in this work. All thermodynamic properties such as pressure, heat capacities, and speed of sound can be calculated from Eq. (4) through differentiation of the Helmholtz energy with respect to temperature and/or density. The expressions for these properties can be found in Jacobsen *et al.* [1] or Setzmann and Wagner [24]. The following properties calculated using Eqs. (6) and (7) for the specified temperature and density are given for validation of user programs: (1) T = 270 K,  $\rho = 11 \text{ mol} \cdot \text{dm}^{-3}$ , p = 27.0621 MPa,  $h = 6517.95 \text{ J} \cdot \text{mol}^{-1}$ , and  $w = 459.222 \text{ m} \cdot \text{s}^{-1}$ ; (2) T = 126.2 K,  $\rho = 11.2 \text{ mol} \cdot \text{dm}^{-3}$ , p = 3.39712 MPa,  $h = 816.780 \text{ J} \cdot \text{mol}^{-1}$ , and  $w = 135.571 \text{ m} \cdot \text{s}^{-1}$ .

## 3. COMPARISONS TO EXPERIMENTAL DATA

The accuracy of the equation of state was determined by comparisons of property values calculated with the equation of state to experimental data. These comparisons are based on the percent deviation in any property, *X*, defined as

$$\% \Delta X = 100 \left( \frac{X_{\text{data}} - X_{\text{calc}}}{X_{\text{data}}} \right). \tag{8}$$

Using this definition, the average absolute deviation is defined as

$$AAD = \frac{1}{n} \sum_{i=1}^{n} \left| \% \Delta X_i \right|, \tag{9}$$

where n is the number of data points. Table III shows the results to selected experimental data sets that were used in fitting the equation of state for p- $\rho$ -T, isochoric and saturation heat capacities, speed of sound, and enthalpy data. Figure 1 shows density deviations calculated using the equation of state from p- $\rho$ -T data measured with single- and two-sinker densitometers.

## 4. UNCERTAINTIES

The new reference equation of state describes the p- $\rho$ -T surface with an uncertainty of less than  $\pm 0.02\%$  in density from the triple point up to temperatures of 523 K and pressures up to 12 MPa and from temperatures of 240 to 523 K at pressures less than 30 MPa. The uncertainty is less than  $\pm 0.02\%$  in pressure in the critical region. Further information is given in Figure 2. In the gaseous and supercritical region, highly accurate speed of sound data are represented within  $\pm 0.02\%$  as shown in Figure 3. The estimated uncertainty for the speed of sound in the liquid phase and at pressures above 30 MPa is  $\pm 0.5\%$  and the estimated uncertainty for heat capacities is  $\pm 1\%$ .

The saturation values can be calculated from the equation of state through the use of the Maxwell criterion. The estimated uncertainties of vapor pressure, saturated liquid density, and saturated vapor density are  $\pm 0.02\%$  for each property. The new formulation yields a reasonable extrapolation behavior up to the limits of chemical stability of nitrogen as indicated by comparison to experimental shock tube data.

## 5. THE CALIBRATION EQUATION

In order to describe the thermodynamic behavior of nitrogen in the so-called "natural-gas region" for instrument calibration purposes, a limited equation of state has been developed. This equation is valid from 270 to 350 K at pressures up to 30 MPa and its general functional form is identical to Eq. (4). For the ideal gas heat capacity used in Eq. (5), the simplified equation

$$\frac{c_p^0}{R} = 3.500571 + 1.115488 \times 10^{-20} T^7 \tag{10}$$

was used, which is valid from 270 to 350 K. The ideal gas Helmholtz energy equation can then be expressed as

$$\alpha^0 = \ln(\delta) + 2.500571 \ln(\tau) - 12.76941 - 0.008137875\tau - 1.0150785 \times 10^{-7} \tau^{-7} . \quad (11)$$

The residual Helmholtz energy contribution to the equation of state is given by

$$\alpha^r(\delta,\tau) = \sum_{k=1}^{10} M_k \delta^{i_k} \tau^{j_k} . \tag{12}$$

The coefficients and parameters of this equation are given in Table IV. The associated equation for pressure is given by

$$p = \rho RT \left[ 1 + \sum_{k=1}^{10} i_k M_k \delta^{i_k} \tau^{j_k} \right].$$
 (13)

The maximum deviation between these equations and Eqs. (6) and (7) is 0.001% in density in the specified range and for the speed of sound, the maximum deviation is 0.002% at pressures less than 7 MPa and 0.006% for higher pressures. Thus, the wide-range equation and the simplified equation can be used alternatively without causing significant inconsistencies. The following properties calculated using Eqs. (11) and (12) for the specified temperature and density are given for validation of user programs: T = 270 K,  $\rho = 11 \text{ mol·dm}^{-3}$ , p = 27.0621 MPa,  $h = 6518.00 \text{ J·mol}^{-1}$ , and  $w = 459.220 \text{ m·s}^{-1}$ .

## **REFERENCES**

- 1. R.T Jacobsen, R.B. Stewart, and M. Jahangiri, *J. Phys. Chem. Ref. Data* 15(2):735 (1986).
- 2. R.T Jacobsen and R.B. Stewart, *J. Phys. Chem. Ref. Data* <u>2</u>(4):757 (1973).
- S. Angus, K.M. de Reuck, B. Armstrong, R.T Jacobsen, and R.B. Stewart,
   <u>International Thermodynamic Tables of the Fluid State Volume 6: Nitrogen,</u>
   International Union of Pure and Applied Chemistry, Chemical Data Series, Oxford,
   Pergamon Press, 20 (1979).
- 4. P. Nowak, R. Kleinrahm, and W. Wagner, accepted for publication in *J. Chem. Thermo*. (1997).
- 5. P. Nowak, R. Kleinrahm, and W. Wagner, accepted for publication in *J. Chem. Thermo*. (1997).
- 6. J. Klimeck, R. Kleinrahm, and W. Wagner, preliminary data, Bochum, Germany (1997).
- 7. A. Fenghour, W.A. Wakeham, D. Ferguson, A.C. Scott, and J.T.R. Watson, *J. Chem. Thermodyn.* 25:831 (1993).
- 8. N. Pieperbeck, R. Kleinrahm, and W. Wagner, J. Chem. Thermodyn. 22:175 (1991).

- 9. M. Jaeschke and H.M. Hinze, Fortschr.-Ber. VDI, Ser. 3, No. 262 (1991).
- 10. W. Duschek, R. Kleinrahm, W. Wagner, and M. Jaeschke, *J. Chem. Thermodyn.* 20:1069 (1988).
- 11. H.J. Achtermann, T.K. Bose, H. Rogener, and J.M. St-Arnaud, *Int. J. Thermophys.* 7(3):709 (1986).
- 12. J.W. Magee, *J. Res. NIST* <u>96</u>(6):725 (1991).
- J.P.M. Trusler and M.F. Costa Gomes, <u>The Speed of Sound in Nitrogen and Ethane</u>,
   Report to Groupe Europeen de Recherches Gaziers (GERG), Imperial College of
   Science, Technology and Medicine, London (1996).
- 14. S.J. Boyes, <u>The Speed of Sound in Gases with Application to Equations of State and</u> Sonic Nozzles, Ph.D. Dissertation, University of London (1992).
- 15. M.B. Ewing and J.P.M. Trusler, *Physica A* <u>184</u>:415 (1992).
- 16. P.J. Kortbeek, N.J. Trappeniers, and S.N. Biswas, Int. J. Thermophys. 9(1):103 (1988).
- 17. G. Owren, private communication, The Norwegian Institute of Technology, Dept. Refrigeration and Air Conditioning, Trondheim, Norway (1995).
- 18. P.G. Grini and G.A. Owren, *J. Chem. Thermodyn.* 29:37 (1997).
- 19. R. Span and W. Wagner, submitted to Int. J. Thermophys. (1997).
- 20. Atomic Weights of the Elements 1993. IUPAC Commission on Atomic Weights and Isotopic Abundances. *J. Phys. Chem. Ref. Data*, 24(4):1561 (1993).
- 21. R. Span, E.W. Lemmon, R.T Jacobsen and W. Wagner, to be submitted to *J. Phys. Chem. Ref. Data*, (1997).
- 22. J.D. Cox, D.D. Wagman, V.A. Medvedev, <u>CODATA Key Values for Thermodynamics</u>, Final Report of the CODATA Task Group on Key Values for Thermodynamics, Hemisphere Publishing Co., New York, (1989).
- 23. U. Setzmann and W. Wagner, *Int. J. Thermophys.* <u>10</u>(6):1103 (1989).
- 24. U. Setzmann and W. Wagner, *J. Phys. Chem. Ref. Data* <u>20</u>(6):1061 (1991).
- 25. C. Tegeler, R. Span, and W. Wagner, VDI Fortschritt-Bericht, Ser. 3, No. 480 (1997).
- 26. R. Span and W. Wagner, J. Phys. Chem. Ref. Data <u>25</u>(6):1509 (1996).

- 27. A. Michels, H. Wouters, and J. DeBoer, *Physica* <u>1</u>:587 (1934).
- 28. A. Michels, H. Wouters, and J. DeBoer, *Physica* <u>3</u>(7):585 (1936).
- 29. J. Saurel, J. des Recherches du CNRS 42:22 (1958).
- 30. G.C. Straty and D.E. Diller, J. Chem. Thermodyn. 12(10):927 (1980).
- 31. B.A. Younglove and R.C. McCarty, J. Chem. Thermodyn. 12:1121 (1980).
- 32. L.A. Weber, J. Chem. Thermodyn. 13:389 (1981).

Table I. Parameters and Coefficients of the Equation of State for Nitrogen, Eq. (6)

k	$N_k$	$i_k$	$j_k$	$l_k$	k	$N_k$	$i_k$	$j_k$	$l_k$
1	0.924803575275	1	0.25	0	19	-0.435762336045x10 <sup>-1</sup>	1	4.0	2
2	-0.492448489428	1	0.875	0	20	-0.723174889316x10 <sup>-1</sup>	2	6.0	2
3	0.661883336938	2	0.5	0	21	0.389644315272x10 <sup>-1</sup>	3	6.0	2
4	$-0.192902649201x10^{1}$	2	0.875	0	22	-0.212201363910x10 <sup>-1</sup>	4	3.0	2
5	-0.622469309629x10 <sup>-1</sup>	3	0.375	0	23	$0.408822981509 \times 10^{-2}$	5	3.0	2
6	0.349943957581	3	0.75	0	24	-0.551990017984x10 <sup>-4</sup>	8	6.0	2
7	0.564857472498	1	0.5	1	25	-0.462016716479x10 <sup>-1</sup>	4	16.0	3
8	$-0.161720005987x10^{1}$	1	0.75	1	26	-0.300311716011x10 <sup>-2</sup>	5	11.0	3
9	-0.481395031883	1	2.0	1	27	0.368825891208x10 <sup>-1</sup>	5	15.0	3
10	0.421150636384	3	1.25	1	28	-0.255856846220x10 <sup>-2</sup>	8	12.0	3
11	-0.161962230825x10 <sup>-1</sup>	3	3.5	1	29	0.896915264558x10 <sup>-2</sup>	3	12.0	4
12	0.172100994165	4	1.0	1	30	-0.441513370350x10 <sup>-2</sup>	5	7.0	4
13	0.735448924933x10 <sup>-2</sup>	6	0.5	1	31	0.133722924858x10 <sup>-2</sup>	6	4.0	4
14	0.168077305479x10 <sup>-1</sup>	6	3.0	1	32	0.264832491957x10 <sup>-3</sup>	9	16.0	4
15	-0.107626664179x10 <sup>-2</sup>	7	0.0	1	33	$0.196688194015x10^2$	1	0.0	
16	-0.137318088513x10 <sup>-1</sup>	7	2.75	1	34	$-0.209115600730x10^2$	1	1.0	
17	0.635466899859x10 <sup>-3</sup>	8	0.75	1	35	0.167788306989x10 <sup>-1</sup>	3	2.0	
18	0.304432279419x10 <sup>-2</sup>	8	2.5	1	36	$0.262767566274x10^4$	2	3.0	

Table II. Parameters of the Gaussian Bell Shaped Terms in the Equation of State for Nitrogen, Eq. (6)

k	$\mathbf{\phi}_k$	$\beta_k$	$\gamma_k$
33	20	325	1.16
34	20	325	1.16
35	15	300	1.13
36	25	275	1.25

Table III. Comparisons of the Equation of State for Nitrogen with Experimental Data

Data Type	Author	No. of Points	Pressure Range (MPa)	Temp. Range (K)	AAD
<i>p</i> -ρ- <i>T</i>	Achtermann et al. [11]	35	1.07-28.7	323-323	0.019
<i>p</i> -ρ- <i>T</i>	Duschek et al. [10]	127	0.50-8.01	273-323	0.004
<i>p</i> -ρ- <i>T</i>	Fenghour et al. [7]	50	3.54-37.0	290-680	0.077
<i>p</i> -ρ- <i>T</i>	Jaeschke and Hinze, Burnett [9]	129	0.22-30.2	273-353	0.010
<i>p</i> -ρ- <i>T</i>	Jaeschke and Hinze, Interferometer [9]	499	0.24-28.7	269-353	0.011
<i>p</i> -ρ- <i>T</i>	Klimeck et al. [6]	264	1.11-30.1	240-520	0.002
<i>p</i> -ρ- <i>T</i>	Michels et al. [27]	56	1.93-8.58	273-423	0.016
<i>p</i> -ρ- <i>T</i>	Michels et al. [28]	147	19.5-300.0	273-423	0.017
<i>p</i> -ρ- <i>T</i>	Nowak <i>et al.</i> [4, 5] <sup>1</sup>	861	0.10-12.0	66-340	0.002
<i>p</i> -ρ- <i>T</i>	Nowak $et al. [4]^2$	285	2.98-5.82	124-136	0.002
<i>p</i> -ρ- <i>T</i>	Pieperbeck et al. [8]	124	0.10-12.1	273-323	0.003
<i>p</i> -ρ- <i>T</i>	Saurel [29]	87	1.01-91.2	423-1074	0.099
<i>p</i> -ρ- <i>T</i>	Straty and Diller [30]	287	0.83-34.8	80-300	0.025
<i>p</i> -ρ- <i>T</i>	Other data <sup>3</sup>	983	0.40-2200.0	248-673	0.224
w	Boyes [14]	112	0.05-6.64	250-325	0.011
w	Ewing and Trusler [15]	100	0.00-0.58	80-373	0.001
w	Kortbeek et al. [16]	134	85.0-1000.0	123-298	0.402
w	Trusler and Costa Gomez [13]	72	0.00-30.1	250-350	0.003
w	Younglove and McCarty [31]	237	0.03-1.51	80-350	0.064
w	Other data <sup>3</sup>	520	0.10-2200.0	64-1001	0.349
$c_v$	Magee [12]	173	$6.1\text{-}31.0^{\dagger}$	66-307	0.624
$c_v$	Weber [32]	61	$10.7\text{-}27.5^\dagger$	91-242	0.402
$c_{sat}$	Magee [12]	102		56-101	0.318

Table III. (Continued)

Data Type	Author	No. of Points	Pressure Range (MPa)	Temp. Range (K)	AAD
$\Delta h$	Grini [18]	19	0.31-15.0	160-240	0.127
$\Delta h$	Owren [17]	42	0.32-15.1	160-240	0.103

Data in the critical region are reported separately <sup>2</sup>Deviations in pressure for data in the critical region

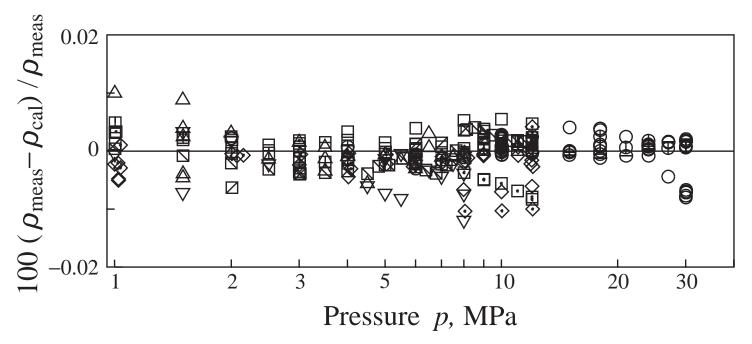
<sup>&</sup>lt;sup>3</sup>Due to space limitations,  $8 p-\rho-T$  and 12 speed of sound data sets have been grouped together as "Other data". References for these data sets are given by Span *et al.* [21] †Density range (mol·dm<sup>-3</sup>)

Table IV. Parameters and Coefficients of the Short Equation of State for Nitrogen, Eq. (11)

k	$M_k$	$i_k$	$j_k$	k	$M_k$	$i_k$	$\dot{J}_k$
1	-0.409226050427	1	-1.0	6	0.112593677045x10 <sup>-1</sup>	3	0.0
2	0.583733818214	1	-0.875	7	$-0.604379290033x10^{-1}$	3	2.875
3	$-0.132040812535x10^{1}$	1	1.625	8	0.567224683248x10 <sup>-2</sup>	4	-0.125
4	0.854602646673x10 <sup>-1</sup>	2	0.125	9	-0.496167879044x10 <sup>-2</sup>	6	-1.0
5	0.207794266769	2	3.5	10	0.572786635566x10 <sup>-2</sup>	6	-0.875

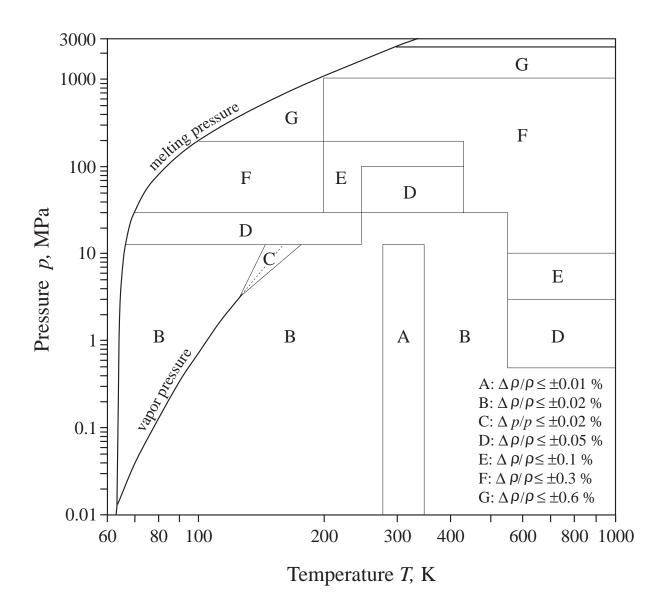
## FIGURE CAPTIONS

- Fig. 1. Comparisons of experimental p- $\rho$ -T data to calculations from the new equation of state for nitrogen.
- Fig. 2. Uncertainties in density for the new equation of state for nitrogen.
- Fig. 3. Uncertainties in speed of sound for the new equation of state for nitrogen.

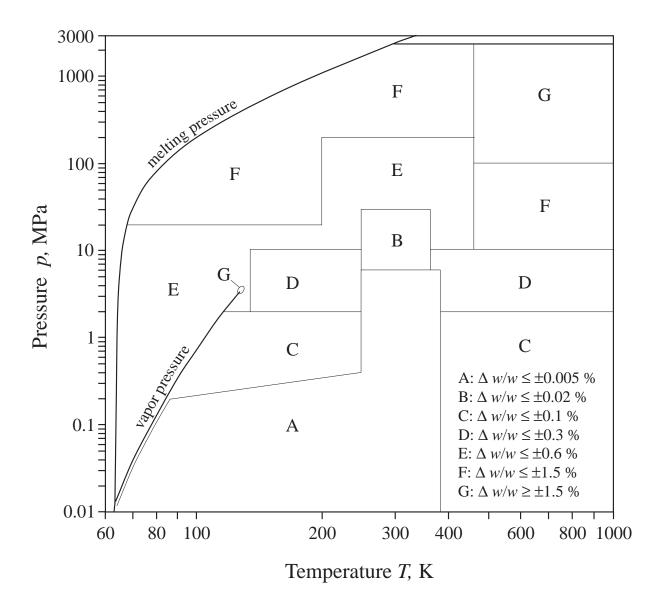


- □ Nowak *et al.* [4], 66 K
- ☑ Nowak *et al.* [4], 110 K
- Nowak *et al.* [4], 240 K
- O Klimeck *et al.* [6], 240 K
- Klimeck *et al.* [6], 520 K
- ♦ Pieperbeck et al. [8], 323 K
- ∇ Duschek *et al.* [10], 323 K

- □ Nowak *et al.* [4], 150 K
- □ Nowak *et al.* [4], 320 K
- ⊖ Klimeck *et al.* [6], 310 K
- ♦ Pieperbeck et al. [8], 273 K
- △ Duschek *et al.* [10], 273 K



Span et al., Fig. 2



Span et al., Fig. 3